ARTICLE

Marni Sheppeard
Mass Matrix Transforms in Qubit Field Theory
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An open letter concerning

Mass matrix transforms in qubit field theory

Marni Sheppeard

To whomever ...

This small paper reports on the initial observation that Carl Brannen's mass operators are naturally expressed as discrete Fourier series, common in the theory of quantum computation. Our obsession with these simple matrices has generated a great deal of criticism. Lubos Motl, the string theorist, called us F–ing Crackpots on my blog, Arcadian Functor, and all attempts to have this paper endorsed for the preprint arxiv failed. Although it is to be recognized that the abundance of errors in much of my writing is regrettable, in my experience these errors are never corrected by the people who think that this work is trivial and wrong, because it would be beneath them to consider it seriously.

The difficulty here is that our motivation for studying these mass operators lies not in standard particle physics, nor in standard theories of gravity, neither of which have anything whatsoever to say about the rest masses of fundamental particles. Unfortunately, a basic idea in quantum field theory is that certain parameters, such as mass, vary continuously, in a very complex way, from singular raw values. An unwavering belief in this idea leads people to conclude that simple formulae for rest masses cannot exist. Of those knowledgeable people willing to consider that such formulae may exist, most appear to believe that one should make no attempt to publish papers about it until one has constructed a complete theory of quantum gravity.

Since Carl Brannen, and many others, have now traveled a fair distance down this road, it seems that a rather impressive theory will actually exist before a single paper is published in a highly regarded peer reviewed journal. Fortunately, thanks to the Internet Age, a rapidly growing number of people are now working on this subject.

Mass matrix transforms in qubit field theory

M. D. Sheppeard
Department of Physics and Astronomy
University of Canterbury,
Christchurch, New Zealand

Circulant mass matrices for triples of charged and neutral leptons have been studied in the context of qubit quantum field theory. This note describes the discrete Fourier transform behind such matrices, and discusses a category theoretic interpretation of these operators.

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INTRODUCTION

Using a measurement algebra approach to QFT, Brannen [1] recently recovered the Koide [2] formula

$$(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = \frac{3}{2}(m_e + m_\mu + m_\tau) \tag{1}$$

for charged lepton masses in the form of a 3×3 circulant complex matrix, whose eigenvalues squared give the lepton masses to experimental precision. This analysis was extended to a set of three neutrinos, and the mass ratio predictions agree with preliminary neutrino oscillation data.

Here it is observed that the discrete Fourier transform [3] provides a further interpretation of the mass matrices, both as a duality between operators and eigenvalues and also as a link to the theory of quantum computation [4].

It is expected that other triples of Standard Model particles, namely baryons and mesons, will also be associated with 3×3 matrix operators of the same kind in accord with their preon structure [1] and the association of spatial directions to the number of particle generations, given by the three primitive idempotents of the measurement algebra.

FOURIER TRANSFORMS AND MASS MATRICES

A *circulant* matrix is built from its first row by adding cyclic permutations. In particular, a 3×3 circulant takes the form

$$\begin{pmatrix} A & B & C \\ C & A & B \\ B & C & A \end{pmatrix} \quad (2)$$

where A, B and C will be complex numbers. Note that any such circulant is a combination of the three permutations (123), (231) and (312). For real eigenvalues λ_k it is essential that A be real and $C = \overline{B}$. Thus a mass matrix [1] takes the form

$$C = \eta \begin{pmatrix} 1 & re^{i\theta} & re^{-i\theta} \\ re^{-i\theta} & 1 & re^{i\theta} \\ re^{i\theta} & re^{-i\theta} & 1 \end{pmatrix} \quad (3)$$

for real η, r and θ. In terms of these parameters, the eigenvalues are given by

$$\lambda_k = \eta(1 + 2r\cos(\theta + \frac{2\pi k}{3}))$$

The Koide formula (1) follows when $r^2 = \frac{1}{2}$ and this choice may be applied also to the neutrino matrix.

In the $n \times n$ case, the discrete Fourier transform [3][4] interchanges the set of eigenvalues λ_k (assumed distinct) and matrix entries $A_1, A_2, A_3, \cdots, A_n$ via

$$\lambda_k = \sum_j e^{\frac{2\pi ijk}{n}} A_j \quad (4)$$

$$A_j = \frac{1}{n} \sum_k e^{-\frac{2\pi ijk}{n}} \lambda_k$$

Viewing the eigenvalues as a diagonal matrix, the transform interchanges the bases of projection operators and cyclic permutations. For real eigenvalues (m_1, m_2, m_3) with $m_i = \lambda_i^2$ in the above, and letting $\omega = e^{\frac{2\pi i}{3}}$, the transform takes the diagonal matrix to the circulant

$$\begin{pmatrix} m_1 + m_2 + m_3 & m_1\omega + m_2\omega^2 + m_3 & m_1\omega^2 + m_2\omega + m_3 \\ m_1\omega^2 + m_2\omega + m_3 & m_1 + m_2 + m_3 & m_1\omega + m_2\omega^2 + m_3 \\ m_1\omega + m_2\omega^2 + m_3 & m_1\omega^2 + m_2\omega + m_3 & m_1 + m_2 + m_3 \end{pmatrix}$$

which must be the square of (3) since the square of a circulant is a circulant. Thus a choice of scale is specified by $\eta = \frac{1}{3}(m_1 + m_2 + m_3)$.

A 3×3 matrix is viewed as a function on the discrete torus $\mathbb{Z}_3 \times \mathbb{Z}_3$, which has a quantum description and a convolution product for matrices [3]. Letting $D_{ij} = \delta_{ij}\omega^i$ there is a Weyl rule

$$D \circ (312) = \omega(312) \circ D$$

where the phase $\frac{2\pi}{3}$ is proportional to $\hbar^{-1}$. This associates Planck's constant with a hierarchy $\mathbb{N}$ determined by the size of the matrix, but the continuum limit is obtained via $\hbar \to \infty$ rather than $\hbar \to 0$.

If masses are to be thought of as quantum numbers, then why are their values so awkward in comparison to, say, spin? For 2×2 circulants with entries A and B, the eigenvectors are $(1, 1)$ and $(1, -1)$ with eigenvalues $(A + B)$ and $(A - B)$ respectively. For example, for the Pauli swap matrix σ_x, with $A = 0$, the spin eigenvalues are ± 1. Complexity in the eigenvalue set only arises in dimension three or higher.

Degenerate eigenvalues $\frac{\lambda_k}{\eta} \in \{1 - r, 1 - r, 1 + 2r\}$ occur when $\theta = 0$ and all matrix entries are real. Although this pattern does not describe the leptons, we observe that it is the typical composition of masses for baryon constituents. Since such mass operators arise in a preon model that unifies particle structure, it is expected that all standard model bound states and resonances may be arranged into mass triples.

In quantum computation [4] a Fourier transform is also defined in this way, acting on a set of n basis states. For example, an N qubit computation uses $n = 2^N$ basis states. The transform is unitary and it may be built from unitary gates, namely the Hadamard gate $H = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$ and the series

$$B_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{2^k}} \end{pmatrix}$$

By analogy, a mass computation with 3^N basis states uses ternary digits, so the gates B_k would be replaced by gates

$$T_k = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3^k}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3^k}} \end{pmatrix} \tag{5}$$

which are also unitary. In general, the Fourier operator entries F_{ij} are given by ω^{ij}, and the theory of *mutually unbiased bases* generalises the Pauli operator algebra in all prime power dimensions.

A basic time evolution operator exists for each dimension n. Note, however, that unlike in conventional constructions, this local evolution is not in any way associated with an emergent cosmic clock, the latter being more closely related to the scale $\hbar$, given here by the matrix dimension. That is, this approach does not assume a globally defined time for a nonsensical universal observer.

DISCUSSION

The mass matrices arise from a one dimensional discrete transform, which itself involves commutative variables. However, it is seen that phase space variables satisfy the Weyl algebra of the quantum plane. Is there a noncommutative transform that extends this analysis to nonclassical underlying spaces? This is relevant to the question of extending the perturbative rest mass computations [1] to nonperturbative regimes.

Kapranov [5] has recently considered path spaces approximated by cubical paths, each of which is represented by a noncommutative monomial in the spatial directions. In dimension $d > 1$ a noncommutative Fourier transform relates measures on the space of paths to functions of the noncommuting variables. The basic idea is that a path integral is just a map from a noncommutative ring to a suitable commutative subring. In this way, particle masses [1] could arise as path integral invariants.

Taking T-duality seriously, one also expects to deal with nonassociativity. From a category theoretic point of view, both noncommutative and nonassociative structures can be dealt with in a unified framework. The cohomological element of interest here is the parity cube axiom, which describes the now familiar pentagon law on five of its faces. In a sufficiently lax algebraic setting, such as for tetracategories, the sixth face may break this law, providing the deformation parameter that turns a pentagon into a hexagon representing the permutation group S_3 [6].

The generation count by primitive idempotents [1] is confirmed by the string theoretic index theorem argument applied to the Riemann moduli space of the six punctured sphere, which has an orbifold Euler characteristic [7] of -6. The six punctures are associated to the six faces of a cube via a dual vertex, which is thickened to a sphere. Note that cohomological integrals for such moduli spaces commonly appear in QFT computations as multiple zeta values and polylogarithms.

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